

Research article

# MODELLING OF TWO DIMENSIONAL FLOW TRANSPORT OF E.COLI AND NITROGEN IN WOJI RIVER, PORT HARCOURT METROPOLIS, NIGER DELTA OF NIGERIA

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## Abstract

Two dimension flow equation were expressed through developed system to monitor the transport process of E.coli and nitrogen. The study were carried out to monitor the deposition of these contaminant at Woji River, base on the condition of the River, several parameters were considered in the system to produces the governing equation for the study. The expressed governing equation were derived considering boundary values that should determined there various limit in their migration process, there derived solution generate various model base on different conditions applied to monitor both parameters, the study is imperative because it will definitely become a useful tools in monitoring and evaluations of both parameters in various conditions. **Copyright ©WJECE, all rights reserved.**

**Keywords:** modeling, two dimensional flows, E.coli, Nitrogen and Woji River

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## 1. Introduction

Their complex biochemical diversity enables them to exist in many different habitats everywhere on earth where they are essential for the geochemical cycle and the elimination of many pollutants. By employing bio-indicators, consisting of organisms or communities of organisms which react to environmental effects by modification of their vital functions, it is possible to draw conclusions on the state of their environment. Due to the complexity of services and resources which are supplied by qualitative and quantitative differentiated ecosystems, it is very difficult to find general indicators that characterize the health of an ecosystem. A rich biodiversity, for example indicates a healthy system, but in some cases it can also be a symptom of disturbance when high amounts of nutrients in an aquatic ecosystem cause enhancement of growth. An indicator has to be relevant and useful. Sulphate-reducing bacteria are a large group of anaerobic organisms that have an important

role in many biogeochemical processes as the sulphur cycle and mineralization of organic matter in anoxic marine and freshwater environments and soil (Sitte *et al.* 2010). The sulphite reducing clostridium group, including lostridium *perfringens*, has been shown to play an important role to assess faecal pollution in sediment ecosystems (Mancini *et al.* 2010, Marcheggiani *et al.* 2008, Marcheggiani *et al.*, 2004). Their presence can be influenced not only by organic matter but also by inorganic contaminants such as heavy metals (Mancini *et al.*, 2011; Mancini *et al.* 2008). *C. perfringens*, gram positive anaerobic spore-forming bacteria of the genus *Clostridium*, that does not carry out a dissimilatory reduction of sulphate, can be used as an alternative indicator for faecal contamination in aquatic ecosystems due to its adaptation to different habitats such as soils, sediments, and sewages. Furthermore, *C. perfringens* presence can be correlated to those of parasitic protozoan and enteric viruses in the water column as *Cryptosporidium sp.*, *Aeromonas sp* and *Giardia*. In addition, the spores produced by *C. perfringens* are extremely resistant to disinfection and the WHO (1996) suggests that their presence in filtered supplies may be an indication of the need for treatment. Although the World Health Organization recommends *C. perfringens* as a useful indicator of fecal pollution in water quality surveys (WHO, 1978), this microorganism has been adopted in Europe exclusively as an additional source of water quality information (Cabelli, 1978; Olivieri, 1982; Rhodes *et al.*, 1999). The sequence of the 16S rRNA gene has been widely used as a phylogenetic marker to study genetic relationships between different strains of bacteria (phylogeny). The analysis of this gene can therefore be considered a standard method for the identification of bacteria at the family, genus and species levels (Woese, 1987; Weisburg *et al.*, 1991; Jeng *et al.*, 2001; Lehner *et al.*, 2004; Raju *et al.*, 2006; Johansson *et al.*, 2006), and has in fact been included in the latest edition of *Bergey's Manual of Systematic Bacteriology* (Bergey's Manual, 2005). A strong correlation between ecosystem health and human health can be demonstrated and many new approaches to monitoring and environmental conduction are possible (Lackey RT, 2001). There are four principle categories of ecosystem functions upon which human health is dependant (De Groot, 1992). The first category contains regulatory functions of ecological processes which deliver water, air, and clean soil through energetic and bio-chemical regulation processes, such as the recycling of organic material. The second category is the supply of space and appropriate substrates for human activities such as cultivation, recreation and living spaces.

## 2. Theoretical background

The deposition of E.coli is of serious concern in River water quality, lots of biological waste injected inside the River are degrading water quality, these conditions were investigated through analysis of water quality to determined the rate of pollution including the extension of pollution increase inside the River, physiochemical investigation carried out in this River shows predominant deposition of E.coli and nitrogen in the Woji River, the deposition of these two parameter are as a result of biological and industrial waste. There are lots of water pollution which is becoming a major concern not only in developed regions including the European Union (EU) thus several developing countries. The amplification of agricultural practices, especially the growing use of fertilizers and pesticides, and the specialization and concentration of crop and livestock production, has had an increasing impact on water quality (Ganoulis and Kiourtsidis 2005). The foremost agricultural water contaminants are nitrates, phosphorus, and Pesticides. Increasing nitrate concentrations intimidate the quality of drinking water, while high insect killer use contributes substantially to indirect emissions of toxic substances. Increasing levels of nitrates and phosphorus in surface waters reduce their ability to support plant and animal

life and make them less attractive for recreation. Controlling water pollution from agriculture and other activities of man has made difficult by its particular nature. In most situations, the activities in various sources generate lot of pollution over a wide area, and its sources are diffuse and difficult to identify. In the area of agricultural activities it also varies unpredictably over time and space, and depends not only on rainfall patterns and the land--slopes and soil characteristics, but also on farmers' land use and crop choices, production techniques, and fertilizer and pesticide use. Farmers' decisions, in turn, are affected by market prices for inputs and outputs, as well as by governments' agricultural support policies. In contrast to many industrial and municipal situations, few pollution treatment alternatives are readily available for installation on farms. Pollution control measures must rely heavily on approaches that affect farmers' land use and production decisions. Thus, agricultural policy, which directly influences these decisions, and environmental policy to control agricultural water pollution need to be coordinated and pursued with the same goals in mind (Ganoulis and Kiourtsidis 2005). Anthropogenic impacts have led to major changes and water management problems during the last decades. Critical water management problems are floods, which frequently extend into the Rhine River during spring time, and loss of habitat for aquatic organisms. Especially changes of the riverbed morphology, implemented to facilitate navigation, have caused deterioration of river habitats. Main problems for drinking water quality can be attributed to nitrate and phosphate leaching from agriculture as well as to emissions of heavy metals, biocides and substances affecting the endocrine system

### 3. Governing Equation

$$K \frac{\partial c}{\partial t} = Dx \frac{\partial^2 c}{\partial x^2} + Dy \frac{\partial^2 c}{\partial y^2} - U \frac{\partial c}{\partial x} - V \frac{\partial c}{\partial y} + Kc \quad \dots\dots\dots (1)$$

The expressed system defined the parameters that has influences the migration of E.coli and nitrogen in Woji River, these system look at longitudinal and vertical dispersion of both parameters, these parameters react with others microbes to pressure the migration of E. coli and nitrogen in the River, these developed the governing equation that will be derived to generate model in various conditions, base on the transport system of both parameters.

We split the above equation with respect to direction of flow and dispersion of contaminants, we have:

$$K \frac{\partial c}{\partial t} = Dx \frac{\partial^2 c}{\partial x^2} - U \frac{\partial c}{\partial x} = -\lambda^2 \quad \dots\dots\dots (2)$$

$$K \frac{\partial c}{\partial t} = Dy \frac{\partial^2 c}{\partial y^2} - V \frac{\partial c}{\partial y} + Kc = -\lambda^2 \quad \dots\dots\dots (3)$$

$$\left. \frac{\partial c}{\partial x} \right|_{x=L} = 0 ; C(o^-) = C(o^+) = Co, x = o \quad \dots\dots\dots (4)$$

Where  $L$  is the distance in  $x$ -direction of the River

$$\left. \frac{\partial c}{\partial y} \right|_{y=o, H} = 0 \quad \dots\dots\dots (5)$$

In other to monitor the rate of these two parameters on migration process, there is need to determined various boundary values, this will express their various limit that are determined on the process of monitoring these parameters in the river. The expression in [4] and [5] shows the boundary conditions that are determined on the transport system of both parameters in the River, these conditions are integrated on the derived solutions applied in monitoring nitrogen and E.coli in Woji River.

Where H is the depth of the River,

The solution for equation (1) is of the form:

$$C(x, y, t) = C(x, t) + C(y, t) \quad \dots\dots\dots (6)$$

We consider equation (2), using Bernoulli's method of separation of variations: -

$$K \frac{\partial c}{\partial t} = Dx \frac{\partial^2 c}{\partial x^2} - U \frac{\partial c}{\partial x} = -\lambda^2$$

$$\Rightarrow KT^1 = Dx X^{11} - UX^1 = -\lambda^2$$

$$\Rightarrow KT^1 = -\lambda^2 \text{ and } DX^{11} - UX^1 + \lambda^2 = 0 \quad \dots\dots\dots (7)$$

$$T = A_1 \ell^{\frac{-\lambda^2}{K}t} \quad \dots\dots\dots (8)$$

$$X = A_2 \ell^{m_1x} + A_3 \ell^{-m_2x} \quad \dots\dots\dots (9)$$

The derived solution in [8] and [9] shows the derived model conditions that monitor the transport system, considering time, the deposition of both parameters at various concentration are monitor at these condition with respect to time of concentration, injection of biological waste are carried out at any time thus there are also frequent degradation of oxygen demand critically affecting the aquatic lives, time of concentration where deoxygenating will definitely take place are expressed in the derived model considering such circumstances. These expression were done in two dimensional flow condition, the derived model in [9] also express the condition with respect to two dimensional flow parameters pressured by time and distances.

$$\text{Where } M_1 = \frac{U + \sqrt{U^2 - 4Dx\lambda^2}}{2Dx} \text{ and } M_2 = \frac{U - \sqrt{U^2 - 4Dx\lambda^2}}{2Dx}$$

We combine equations (8) and (9), to have

$$C(x, t) = A_1 \ell^{\frac{-\lambda^2}{K}t} (A_2 \ell^{m_1x} + A_3 \ell^{-m_2x}) \quad \dots\dots\dots (10)$$

Subject (10) to equation (4), we have

$$C(x, t) = \frac{CoM_2 \ell^{M_1L}}{M_2 \ell^{M_2L} + M_1 \ell^{M_1L}} \ell^{\frac{-\lambda^2}{K}t} \left[ \ell^{M_1x} + \frac{M_1 \ell^{M_1L}}{M_2 \ell^{M_2L}} \ell^{M_2x} \right] \quad \dots\dots\dots (11)$$

The expression monitoring the concentration with respect to time and distances between [10] and [11] put together express the rate of pressure that change the concentration, these parameters are integrated in derived solution considering their various function in the system, velocity of flow including vertical and longitudinal dispersions are expression through quadratic equation, and the reactions between these parameters produce exponential phase of the transport, therefore both parameters at these stage are monitored base on the pressure of these parameter considered in the migration process.

$$K \frac{\partial c}{\partial y} = Dy \frac{\partial^2 c}{\partial y^2} - V \frac{\partial c}{\partial y} + Kc = -\lambda^2$$

$$\Rightarrow KT^1 - \lambda^2 \text{ and } DyY^{11} - VY^1 + (Kc + \lambda^2) = 0 \quad \dots\dots\dots (12)$$

$$T = A_1 \ell^{\frac{-\lambda^2}{K}t} \quad \dots\dots\dots (13)$$

The expression splited some parameter in other to monitor the transport system when some of the parameter are not active, for example vertical dispersion, looking the system with present rate of deoxygenating of the River when the contaminant are injected and there velocity of flow on natural condition are at optimum level, the time of deoxygenating with respect to longitudinal dispersions were considered, it also produced exponential phase of the transport system for both parameters.

$$Y = B_1 \text{Cos } n_1 y + B_2 \text{Sin } n_2 y \quad \dots\dots\dots (14)$$

So that, we combine equations (13) and (14) yield

$$C(y,t) = A_1 \ell^{\frac{-\lambda^2}{K}t} (B_1 \text{Cos } n_1 y + B_2 \text{Sin } n_2 y) \quad \dots\dots\dots (15)$$

Subject equation (15) to equation (5), yield: -

$$C(y,t) = \frac{CoM_2 \ell^{M_2L}}{M_2 \ell^{M_2L} + M_1 \ell^{M_1L}} \ell^{\frac{-\lambda^2}{K}t} \left[ b \text{Cos } \frac{n\pi}{H} y \right], n=1,2,3 \quad \dots\dots\dots (16)$$

Summary over a Fourier series, in the section [O, H], we have

$$C(y,t) = \frac{CoM_2 \ell^{M_2L}}{M_2 \ell^{M_2L} + M_1 \ell^{M_1L}} \ell^{\frac{-\lambda^2}{K}t} \left[ \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \text{Cos } \frac{n\pi}{H} y \right] \quad \dots\dots\dots (17)$$

$$\Rightarrow \frac{a_o}{2} + \frac{1}{H} \int_0^H f(u) du, \quad b_1 \ell \frac{2}{H} \int_0^H f(u) \text{Cos } n u du$$

$$\text{Hence, } C(y,t) = \frac{1}{H} \int_0^H f(u) du + \frac{2}{H} \sum \text{Cos } \frac{n\pi}{H} y \int_0^H f(u) \text{Cos } n u du \quad \dots\dots\dots (18)$$

Hence  $b_o = a_o$  and  $b_n = a_n$ , for  $n \geq 1$ .

At this point, the pollutant is following with a uniform underground velocity with transverse dispersion.

Substituting equation (11) and (18) into equation (6), so that we have: -

$$C(x, y, t) = \frac{CoM_2\ell^{M_2L}}{M_2\ell^{M_2L} + M_1\ell^{M_1L}} \ell^{\frac{-\lambda^2}{K}t} \left[ \ell^{M_1x} + \frac{M_1\ell^{M_1L}}{M_2\ell^{M_2L}} \ell^{M_2x} + bn \cos \frac{n\pi}{H} y \right] \dots\dots\dots (19)$$

As  $x \rightarrow \infty$ ,  $y \rightarrow 0$ ,  $c \rightarrow 0$  so that we have

$$C(x, y, t) = \frac{CoM_2\ell^{M_2L}}{M_2\ell^{M_2L} + M_1\ell^{M_1L}} \ell^{\frac{-\lambda^2}{K}t} \left[ \ell^{M_1x} + bo + \sum_{n=1}^{\infty} bn \cos \frac{n\pi}{H} y \right] \dots\dots\dots (20)$$

The expression here defined the behaviour of the both parameters in the River, these conditions are monitored in various dimensions through several variables that pressured the transport process of the contaminant in various ways. The deposition of nitrogen and E.coli were investigated to generate predominant contaminant in the study area, these call for serious concern to critically monitor the behaviour and the migration process including their growth rate, thus through the deposition of nitrogen in the system. The derived solution were able to express these condition integrated in the system as variable, these has definitely analyze ways it pressure the transport system of both parameters. The developed models will definitely express the rate of contaminant for both parameters.

#### 4. Conclusion

Woji River has been contaminant by biological waste of every sort, these condition has degrade lots of aquatic live in the River, lots of studies has been carried out, but there has not been any study carried on the deposition of E.coli and nitrogen in the study area, this environmental unfriendly of these such should be eradicated in other to improve the quality of Woji river and save the aquatic live in marine environment. The developed mathematical model to monitor the transport process of both parameters were to critically express in various condition, both contaminants behaviour were mathematically model in other to determine way the contaminant can reduced it to the harmless level. The derived solution generated model considering various conditions developed from the system, these will definitely produces simulated parameter on the process of monitoring them in various determined conditions.

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